PAPER CODE NO. **PHYS393**	EXAMINER: Dr. K. M. Hock DEPARTMENT: Physics	TEL. NO. 01925-603853

JANUARY EXAMINATIONS 2011

Bachelor of Science: Year 3
Master of Physics: Year 3
Master of Physics: Year 4

STATISTICAL AND LOW TEMPERATURE PHYSICS

TIME ALLOWED: 3 hours

INSTRUCTIONS TO CANDIDATES

Answer all questions.

Question 1 carries 50% of the total marks.

Questions 2 and 3 each carry 25% of the total marks.

Answer either part (a) or part (b) of questions 2 and 3.

In the event of a student answering both parts of an either/or question and not clearly crossing out one answer, only the answer to part (a) of the question will be marked.

The marks allotted to each part of a question are indicated in square brackets.

All symbols have their usual meanings unless otherwise stated.

Question 1.

(a) A salt in magnetic field contains N spin ½ ions at temperature T. The magnetic energy levels for each of the ions are $-\varepsilon$ and ε.

i) Write an expression for the population of each level in terms of N, ε, and T. **[2]**

ii) Show that the internal energy can be written as

$$U = -N\varepsilon \frac{\exp(\varepsilon / k_B T) - \exp(-\varepsilon / k_B T)}{\exp(\varepsilon / k_B T) + \exp(-\varepsilon / k_B T)}. \quad \textbf{[2]}$$

iii) Find the low and high T limits of U. **[2]**

iv) Sketch the variation of U, and heat capacity C, with T. **[2]**

(b) A 1 m^3 box contains one mole of helium at room temperature.

i) Find the average kinetic energy of the atoms. **[2]**

ii) Find the corresponding wavevector. **[2]**

iii) The total number of states with wavevector below k is $G(k) = 4Vk^3/3\pi^2$, where V is the volume. Find the number of states below the average kinetic energy of the atoms. **[2]**

iv) Estimate the probability that one of these states is occupied. **[2]**

v) How does this affect the most probable macrostate? **[2]**

(c) A system of particles, at temperature T, occupies a set of energy states. The probability that a state at energy ε is occupied, is $f(\varepsilon)$.

i) Write down the expressions for $f(\varepsilon)$ if the particles are fermions, and if they are bosons. **[2]**

ii) Why are they different? **[2]**

iii) Sketch a diagram to show how the energy levels of fermions are occupied near 0 K. **[2]**

iv) Sketch a diagram to show how the energy levels of bosons are occupied near 0 K. **[2]**

(d)

i) The presence of a magnetic field in a macroscopic wavefunction of electrons must produce a current. How does this explain the Meissner's effect? **[2]**

ii) Discuss how this gives rise to London's penetration depth. **[3]**

iii) Sketch the heat capacity versus temperature graph for a superconductor, above and below the transition temperature. How does this suggest the existence of an energy gap? **[3]**

(e) A body can move with zero resistance through superfluid ^{4}He.

i) What excitations are possible in superfluid ^{4}He? **[2]**

ii) Consider an excitation of energy E and momentum p. The body's velocity must be above E/p before excitation is possible. What are the conservation laws that lead to this? **[2]**

iii) Sketch the dispersion relation of the excitations in superfluid ^{4}He. Draw the line with gradient equal to the minimum E/p. **[2]**

iv) Why does the body experience no resistance when its velocity is below the minimum E/p? **[2]**

(f)

i) State the electronic, nuclear and total angular momenta of ^{3}He and ^{4}He atoms. **[2]**

ii) Explain why one is a fermion and the other is a boson. **[2]**

iii) The atoms in liquid ^{3}He at 2 K occupy the energy levels in a certain way. Sketch a picture to show this. **[2]**

iv) The atoms in liquid ^{4}He at 2 K occupy the energy levels in a certain way. Sketch a picture to show this. **[2]**

Question 2. Answer **either** (a) **or** (b)

(a) One mole of copper is at 1 K. Each atom supplies two conduction electrons.

i) Assuming that the electrons behave like an ideal gas, write down the expression for the average kinetic energy of the electrons. Hence find the heat capacity. **[3]**

ii) At 1 K, the measured heat capacity is 0.6 mJ/K. Explain why it is different. **[2]**

iii) With the help of a graph, estimate the energy range of the electrons that are excited above the Fermi energy at temperature T. **[4]**

iv) Using the density of states for the ideal gas,

$$g(\varepsilon) = \frac{4m\pi V}{h^3}\sqrt{2m\varepsilon}\ ,$$

derive an expression for the number, n, of excited electrons. (Molar volume of copper is 7.11 cm^3). **[4]**

v) Why is it reasonable to suppose that these electrons behave like the ideal gas? **[4]**

vi) Derive an expression for the heat capacity using the ideal gas assumption. **[4]**

vii) Find the Fermi energy. Calculate the heat capacity for copper at 1 K. Compare with the measured value and comment. **[4]**

(b) N conduction electrons move freely inside a cube of metal of side L.

i) Write quantised values for the wavevector components k_x, k_y, k_z. **[2]**

ii) Illustrate the allowed states in k space. **[2]**

iii) Show that the number of states with wavevectors in the range k to k+dk is

$$g(k)dk = \frac{2Vk^2.dk}{\pi^2},$$

where V is the volume L^3. **[6]**

iv) Write the relation between k and energy ε. **[1]**

v) Show that the number of states with energy in the range ε to ε+dε is

$$g(\varepsilon) = \frac{2mV}{\pi^2\hbar^3}\sqrt{2m\varepsilon}\ .$$ **[3]**

vi) The probability that a state with energy ε will be occupied is f(ε). Sketch graphs of f(ε) versus ε for electrons for temperature T = 0 K, and for nonzero T much less that the Fermi temperature T_F. Indicate the Fermi energy E_F. **[3]**

vii) Show that E_F is given by

$$E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}.$$ **[5]**

viii) Silver has a molar volume of 10.27×10^{-6} m^3. Each atom contributes one conduction electron. Find the Fermi energy. **[3]**

Question 3. Answer **<u>either</u>** (a) **<u>or</u>** (b)

(a)

i) Why is the Bose Einstein condensate a good candidate for explaining superfluidity and superconductivity? **[5]**

ii) Using a sketch of the Fermi Dirac distribution graph, explain what happens to the chemical potential μ of a boson gas as temperature falls to 0 K? **[5]**

iii) In terms of the density of states g(ε), write down the expression for the number of particles N. Explain when and why the number of excited bosons may be written as

$$N_{ex} = \int_0^\infty \frac{g(\varepsilon)d\varepsilon}{\exp(\varepsilon / k_B T) - 1}.$$ **[5]**

iv) Given that the solution is

$$N_{ex} = \left(\frac{2\pi m k_B T}{h^2}\right)^{3/2} 2.612V,$$

explain how to find the condensation temperature T_{BE}. Find the value for liquid ^{4}He, with molar volume 36.84 cm^3. **[5]**

v) In terms of g(ε), write down the expression for the energy U below T_{BE}. Given that

$$U = 0.7704 k_B N \frac{T^{5/2}}{T_{BE}^{3/2}}$$

is the solution, derive the heat capacity C. Calculate the value of C for one mole of ^{4}He at T_{BE}, and sketch the graph of C versus T. **[5]**

(b)

i) Describe qualitatively the basic features of the theory of superconductivity. **[5]**

ii) Explain qualitatively what happens as the temperature of a superconductor rises above the critical temperature. **[2]**

iii) Explain qualitatively what happens as a magnetic field higher than the critical field is applied to a superconductor. **[2]**

iv) Describe the isotope effect in superconductivity. What does it prove? **[4]**

v) Describe the Meissner effect. **[5]**

vi) Give two main characteristics of high-T_C superconductor materials. **[2]**

vii) How does the magnetic flux quantum confirm the BCS theory? Mention one important application of magnetic flux quantization. **[5]**

CONSTANTS

Speed of light in vacuum	c	=	3.00×10^{8} ms^{-1}
Permeability of vacuum	μ_0	=	$4\pi \times 10^{-7}$ Hm^{-1}
		=	$4\pi \times 10^{-7}$ VsA^{-1}m^{-1}
Permittivity of vacuum	ε_0	=	8.85×10^{-12} Fm^{-1}
		=	8.85×10^{-12} AsV^{-1}m^{-1}
Elementary charge	e	=	1.60×10^{-19} C
Planck constant	h	=	6.63×10^{-34} Js
	$h/2\pi = \hbar$	=	1.05×10^{-34} Js
Avogadro constant	N_A	=	6.02×10^{23} mol^{-1}
Boltzmann constant	k_B	=	1.38×10^{-23} JK^{-1}
Gas constant	R	=	8.31 JK^{-1}mol^{-1}
Unified atomic mass constant	m_u	=	1.66×10^{-27} kg
		=	931.5 MeVc^{-2}
Electron mass	m_e	=	9.11×10^{-31} kg
Proton mass	m_p	=	1.67×10^{-27} kg
Gravitational constant	G	=	6.67×10^{-11} Nm^{2}kg^{-2}
Acceleration due to gravity	g	=	9.8 ms^{-2}
Bohr magneton	μ_B	=	9.27×10^{-24} JT^{-1}